EFFICIENT COMPUTATION OF DESIGN DERIVATIVES FOR AEROELASTIC SYSTEMS

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ABSTRACT

Introduction

With the aviation industry striving to reduce emissions, higher aspect ratio planforms are of considerable interest due to their reduction in lift-induced drag. The aeroelastic behaviour of such configurations must be characterised to ensure no harmful instabilities occur within the flight envelope, which can be done through coupled aeroelastic simulation. For medium-fidelity aeroelastic analysis, time-domain aerodynamic forcing is generally predicted with panel-based potential flow aerodynamic methods, with such methods being valuable due to their relatively low computation expense. These can be coupled with nonlinear structural beam models which prove suitable for modelling high aspect ratio wings experiencing large deformation. This allows for a range of static and dynamic analysis for arbitrary aircraft configurations, such as that available in SHARPy [1].

Aircraft design is largely driven by efficiency, and as such, any design optimization will usually focus on reducing drag or structural mass. However, it is important that optimized designs do not encounter problematic aeroelastic instabilities in the flight envelope. If we wish to encourage specific aeroelastic behaviours in a gradient-based optimization, we require gradients of the output values of interest, such as structural loads or the flutter velocity, with respect to our design variables, such as mass distribution or the wing chord distribution. This allows for gradient-based optimization to be performed on some loss function of the system output with respect to these design parameters. Existing work has obtained UVLM sensitivities [2] and nonlinear beam structure sensitivities [3].

Problem Statement

We seek to form methods for the integration of time-domain aeroelastic analysis and flutter on the design optimization of flexible aircraft configurations. For gradient-based methods, this requires derivatives of the coupled aerodynamic and structure problem with respect to design parameters. We are interested in obtained gradients for the time-domain response limit cycle oscillations (LCOs). By time stepping the solution from some perturbed initial condition until a limit cycle is achieved, analysis of this cycle can be performed. Alternatively, if we would prefer to remove all instabilities, we can use linearised aeroelastic methods which reduce the computational burden considerably, as no time domain solution is required. Such linearized models can be formed around some reference deformed [4] and can predict both flutter and divergence.

We propose an implementation using the Google JAX machine learning framework, which allows for vectorisation, GPU acceleration, as well as automatic differentiation, with this having been leveraged in the existing FENIAX aeroelastic codebase [5]. By implementing the numerics in this framework, we can obtain the Jacobian of the desired output data time series, which may include for example the total forcing or the circulation distribution, with respect to the geometric parameters. For uses in real-world problems, we will have some objective to minimize in design, which may, for example, aim to minimize the maximum strain observed in the beam; this would be described with a loss function, which can be minimised using gradient descent methods. For the linear case, we seek to obtain derivatives of the matrices which describe the linear model, with respect to design parameters; in turn, derivatives of the stability eigenvalues of the system are to be found.

Results

As a motivating example of part of the nonlinear solution, we introduce the case of a rigid planar wing with two free ends experiencing harmonic pitching and plunging motion, with reduced frequency of 0.1, amplitude to chord ratio of 0.25, pitch amplitude of 4°, and aspect ratio 8. An oscillatory case was chosen; in turn, it allowed for investigation of which planform shapes maximize lift when undergoing harmonic motions. This case is started from a steady state solution and run until the forcing is approximately harmonic. Aerodynamics are modelled using the UVLM with a prescribed trailing wake; depictions of this wing at rest and during oscillations are presented in Figure 1.

From the resulting solution, we choose our loss function to be the maximum lift force observed during the response. In turn, this results in the gradient being obtained for the point with the greatest magnitude only. Here we choose to find derivatives with respect to the length of the chord and the fraction along the chord where we prescribe motion. These derivatives obtained for the time series using AD and compared against finite difference (FD) evaluations using a perturbation of 10^{-6} are provided in Table 1. It can be seen from these results that for both design variables, the AD and FD results are almost identical, with the error attributed to the size of perturbation.

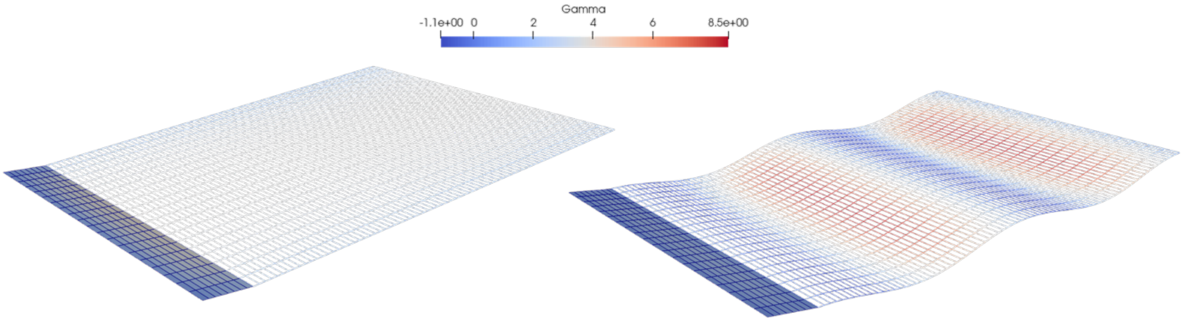

Figure 1 - Wing at initial steady state (left), and wing at final timestep after undergoing oscillations (right). The wing (solid) and trailing wake (wireframe) are coloured with circulation strengths.

Table 1 – Derivatives of the maximum transient lift with respect to design variables.

Parameter	**Automatic Differentiation Derivative**	**Finite Difference Derivative**
Chord length	395.7892	395.7895
Prescribed motion chord fraction	110.9697	110.9700

Future Work

For the final paper, we intend to:

- Integrate the UVLM with a differentiable, geometrically exact beam structural solver.
- Formulate and implement a linearized aeroelastic system that can be differentiated with respect to design variables.
- Formulate loss functions for given aeroelastic design objectives.
- Demonstrate the ability to optimize the flutter and time-domain aeroelastic behaviour for an example configuration, here to be chosen as the Pazy wing.

References

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