AN EFFICIENT GEOMETRICALLY NONLINEAR AEROELASTIC FRAMEWORK FOR GRADIENT-BASED AEROELASTIC OPTIMIZATION

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Introduction

The pursuit of higher aerodynamic efficiencies has driven higher aspect-ratio wing designs, which in turn are subject to larger aeroelastic deformations during flight. As a result, future aircraft may require nonlinear aeroelastic analyses, ideally early in the design cycle/multidisciplinary design optimization (MDO) process. Flutter, in particular, must be considered during design to avoid late stage defects that are expensive and time consuming to remedy. An exhaustive review of flutter and post-flutter constraints for MDO was compiled by Jonsson et al. [1].

Lupp and Cesnik [2] first introduced a method to determine gradients for static and dynamic aeroelastic analyses (flutter) that use a strain-based, geometrically nonlinear beam formulation. The approach built on the strain-based formulation by Cesnik and co-workers [3–8] and utilized Automatic Differentiation (AD) to obtain gradients from UM/NAST via operator overloading. They observed that AD alone resulted in a substantial performance penalties, requiring mitigation strategies. This approach has since been used and extended by Rosatelli et al. [9, 10] to augment gradient-based MDO problems with geometrically nonlinear static (strength) and dynamic aeroelastic (free-flight flutter) constraints.

This paper will present improvements to the strain-based nonlinear beam formulation and introduce analytical derivatives, providing efficient methods for including gradient-based, geometrically nonlinear aeroelastic optimization constraints. A hallmark of the proposed approach, is its modularity, allowing the integration of a variety of aerodynamic models while preserving the nonlinear, coupled nature of the aeroelastic analysis and the ability to obtain coupled aeroelastic sensitivities.

Strain-Based Beam Formulation

A geometrically nonlinear, constant strain, three-noded beam element is used within this work. The strain states for every beam element are: extensional, twist, and two bending curvatures (in-plane and out-of-plane):

$$\varepsilon^{el} = \begin{bmatrix} \varepsilon_x \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}. \quad (1)$$

0

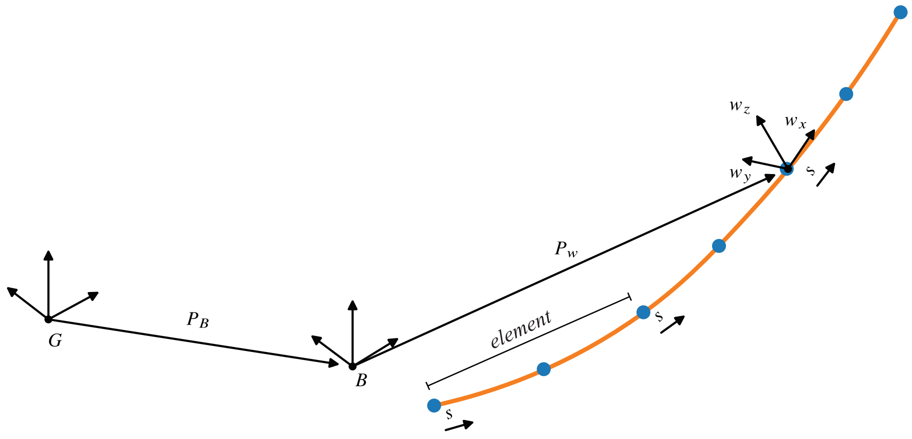

Figure 1: Global, body, and nodal/local coordinate system definitions used in the Perseids aeroelastic toolbox and UM/NAST theoretical formulation.

A column vector h can be defined that contains a point's spatial position and orientation information as a function of the element coordinate, s:

$$h\left(s\right)=\begin{bmatrix}P_b+P_w(s)\\ w_x(s)\\ w_y(s)\\ w_z(s)\end{bmatrix}. \tag{2}$$

The kinematic relationship between the strain (state variable), the boundary node h_{BC}, and the beam displacements (represented in the vector h) is captured by the matrix exponential:

$$h_w(s)=e^{\mathbb{K}(s-s_0)}h_{BC} \tag{3}$$

with

$$\mathbb{K}\left(s\right)=\begin{bmatrix}0 & 1+\varepsilon_x & 0 & 0\\ 0 & 0 & \kappa_z & -\kappa_y\\ 0 & -\kappa_z & 0 & \kappa_x\\ 0 & \kappa_y & -\kappa_x & 0\end{bmatrix}_{12\times 12}. \tag{4}$$

Furthermore, the formulation provides Jacobians (e.g., $J_{h\varepsilon}$, relating the displacement frame to strains) to convert between different frames of reference. For further details on the strain-based formulation, the reader is encouraged to review the comprehensive description provided by Su and Cesnik [7, 11].

The steady state equations of motion were determined by Su and Cesnik [7, 11]:

$$\underbrace{\begin{bmatrix}K_F & K_c^T\\ K_c & 0\end{bmatrix}}_{K_k}\underbrace{\begin{bmatrix}\varepsilon\\ \lambda_c\end{bmatrix}}_{y_{s,k}}=\underbrace{\begin{bmatrix}f\\ f_c\end{bmatrix}}_{f_k}. \tag{5}$$

This can be reformulated into residual form, including a relaxation factor to aid numerical convergence [12],

$$\mathcal{R}_{s,k} = \zeta_n(k)\, J_{p\varepsilon}^T f - K_k y_{s,k}. \tag{6}$$

where ζ_n is the numerical relaxation factor, $J_{p\varepsilon}$ is the Jacobian that relates quantities from displacements to strains.

Preliminary Results

The preliminary results to this paper illustrate the work conducted to analytically derive the partial derivatives needed to apply the adjoint method [13] to the strain-based, geometrically nonlinear beam formulation. The adjoint solution is obtained by first determining df/dr from the linear system in Equation 7 and substituting into 8:

$$-\left[\frac{\partial \mathcal{R}}{\partial y}\right]^T \left[\frac{dy}{dx}\right]^T = \left[\frac{\partial F}{\partial y}\right]^T \tag{7}$$

$$\frac{df}{dx} = \frac{\partial F}{\partial x} + \frac{df}{dr}\frac{d\mathcal{R}}{dx} \tag{8}$$

For the unconstrained beam, the residuals only contain strain states (following Equation 5). The partial derivatives of the residual equations w.r.t. the strain states is:

$$\frac{\partial \mathcal{R}_s}{\partial \varepsilon} = \frac{\partial\left(J_{p\varepsilon}^T f_k\right)}{\partial \varepsilon} - \frac{\partial K_F}{\partial \varepsilon}\varepsilon - K_k. \tag{9}$$

The element flexible stiffness matrix consists of:

$$K_{F,e} = \begin{bmatrix} EA & k_{12} & k_{13} & k_{14} \\ k_{12} & GJ & k_{23} & k_{24} \\ k_{13} & k_{23} & EI_{yy} & k_{34} \\ k_{14} & k_{24} & k_{34} & EI_{zz} \end{bmatrix} l_e \tag{10}$$

However, the element flexible stiffness matrix does not depend on the strain state, and the assembly stiffness matrix is a block-diagonal matrix containing the element stiffness contributions. This results in the partials of the entire stiffness matrix associated with flexible states to simplify to:

$$\frac{\partial K_F}{\partial \varepsilon} = 0. \tag{11}$$

Considering only an unconstrained system, the partials of the residual equations w.r.t. the strain states is:

$$\frac{\partial \mathcal{R}_s}{\partial \varepsilon} = \frac{\partial\left(J_{p\varepsilon}^T f_k\right)}{\partial \varepsilon} - K_k \tag{12}$$

$$= \frac{\partial J_{p\varepsilon}^T}{\partial \varepsilon} f_k + J_{p\varepsilon}^T \frac{\partial f_k}{\partial \varepsilon} - K_k. \tag{13}$$

As the applied loads are not a function of the strain states, this simplifies to:

$$\frac{\partial \mathcal{R}_s}{\partial \varepsilon} = \frac{\partial J_{p\varepsilon}^T}{\partial \varepsilon} f_k - K_k. \tag{14}$$

Similar derivations have been conducted for the partial derivatives of the residual equations w.r.t. the design variables, as well as for a function of interest (aggregated strength constraint) and will be presented in the final paper. Coupled aeroelastic sensitivities will be determined using the Unified Derivative Equations [14], which are a generalization of the adjoint equations.

Final Paper

This abstract presented a nonlinear beam formulation originally derived by Su and Cesnik and extended it to obtain analytical gradients for efficient inclusion of aeroelastic analyses into MDO processes. While past work by Lupp and Cesnik have implemented gradients in nonlinear beam analyses for aeroelastic simulations using Automatic Differentiation, the work presented in this abstract enables significant efficiency gains in terms of execution time and memory requirements compared to previous work. The presented approach to determining gradients is also inherently accurate, as they are analytically derived and therefore numerically exact.

The final paper will expand on the analytical gradient derivations to consider cases that feature relative and absolute displacement constraints. Additionally, modular and efficient approaches to load and displacement transfer and their derivatives will be presented, completing the fully coupled, nonlinear aeroelastic formulation including derivatives for gradient-based optimization. Finally, efficiency benchmarks and gradient-based MDO optimizations of a notional aircraft will be presented that utilize the proposed formulation to illustrate its application within large-scale MDO problems.

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