EIGENVALUE SENSITIVITY ANALYSIS FOR AEROSERVOELASTIC CONTROL CO-DESIGN

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ABSTRACT

Background: A key consideration in the design of an aircraft is the susceptibility to flutter. By optimizing the structure of the aircraft concurrently with the implementation of an active flutter suppression (AFS) system, significant structural weight otherwise necessary to prevent flutter, can be removed from an aircraft [1]. However, co-design of AFS and aircraft structure requires knowledge of how changes in the design parameters affect the closed-loop dynamics of the system. This often requires using computationally expensive and inaccurate finite difference methods, or the use of slower, gradient-free optimization algorithms.

In this work, we demonstrate the application of an efficient method for eigenvalue sensitivity analysis of systems with closed loop control, applied to the gradient-based aeroservoelastic control co-design optimization of a 600 mm long flexible wing model.

Sensitivity Analysis: We demonstrate a method of eigenvalue sensitivity analysis that can be applied to any co-design problem involving a controller derived via an algebraic Riccati equation, such as LQR, LQG, H_2 and H_∞. The key challenge in extracting sensitivities of the closed-loop dynamics is computing the derivative of the solution of the algebraic Riccati equation, P, resulting from the LQR optimization problem with respect to the design parameters, $\boldsymbol{\theta}$: $\frac{\partial P}{\partial \boldsymbol{\theta}}$. By taking the derivative of the Riccati equation and reformulating it as a Lyapunov equation for $\frac{\partial P}{\partial \boldsymbol{\theta}}$, we can solve for the exact closed-loop eigenvalues efficiently using standard numerical libraries, eliminating the need for finite difference methods.

ASE Modeling: This analysis is applied to the design optimization of a flexible wing with AFS based on the Pazy wing---a benchmark high aspect ratio wing designed for studying nonlinear aeroelastic phenomena at large deflections [2]. Most of the stiffness and mass of the wing is accounted for in its central spar. This allows the construction of a low-order model that is sufficiently accurate at low deflections and captures the flutter characteristics of the wing.

The aeroservoelastic model is a simple finite element model, coupled to a quasi-steady aerodynamic model. The mass and stiffness of each element is calculated from the dimensions and material properties of the central spar, plus any additional structural or added mass. Where a control surface is present, a forcing term is added that is proportional to the angle of the control surface. Modal damping ratios are computed using the eigenvalues and eigenvectors of the complete state-space representation ($\boldsymbol{x} = A\boldsymbol{x} + B\boldsymbol{u}$).

This model is completely parametric, allowing for significant flexibility in the design of the central spar as well as placement, size, and number of control surfaces. The analytical formulation of the model makes computation of the model's sensitivity to different design parameters straightforward.

Optimization and Results: To demonstrate the eigenvalue sensitivity computation's efficacy in aeroservoelastic control co-design, a few optimization problems are posed with the goal of minimizing the structural mass of the wing while maintaining the flutter characteristics of the uncontrolled wing. Results utilizing an epsilon-constraint formulation are shown in Figure 1.

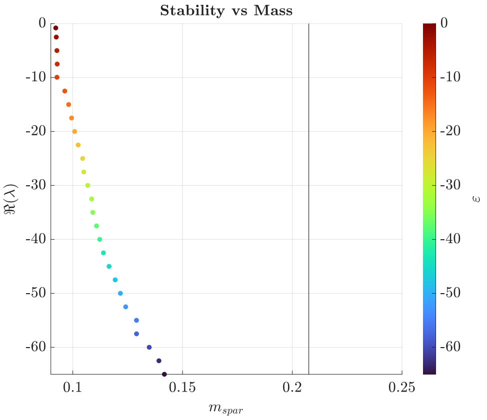

Figure 1: Epsilon-constraint optimization

The optimization achieved up to an approximately 35% reduction in total wing mass compared to the uncontrolled wing, while maintaining the same, or higher, critical flutter speed. Computational cost of optimizations was significantly reduced compared to using finite-difference derived sensitivities. Analysis of the locally optimal solutions using K-means clustering, shown in Figure 2, reveals that flutter occurs through distinct mode shapes depending on the design region, with three dominant flutter mode shapes identified across the design space, providing designers with insight into different structural-aerodynamic coupling mechanisms.

Conclusions: This work demonstrates that analytical eigenvalue sensitivity analysis for Riccati-based controllers enables the efficient gradient-based optimization of aeroservoelastic systems, facilitating the integration of active flutter suppression into early-stage aircraft design. The efficiency of the method allows for extensive exploration of design spaces that would be impractical when using finite-difference methods. This supports the development of lighter, more efficient aircraft with extended safe flight envelopes. The approach is applicable to higher fidelity models than used in this work, and any Riccati-based optimal and robust control design methods such as LQR, LQG, H_2 and H_∞.

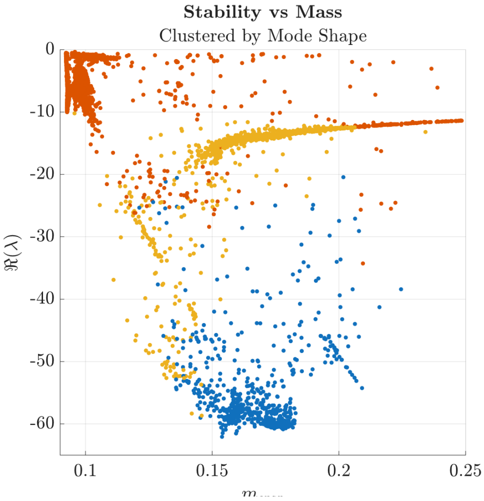

Figure 2: K-means clustering of locally optimal solutions

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[1] Livne, Eli. “Aircraft active flutter suppression: State of the art and technology maturation needs.” Journal of Aircraft, Vol. 55. 1: pp. 410–450. 2018. American Institute of Aeronautics and Astronautics Inc. DOI 10.2514/1.C034442.

[2] Avin, Or, Raveh, Daniella E., Drachinsky, Ariel, Ben-Shmuel, Yaron and Tur, Moshe. “Experimental Aeroelastic Benchmark of a Very Flexible Wing.” AIAA Journal Vol. 60 No. 3 (2022): pp. 1745–1768. DOI 10.2514/1.J060621.