AEROTHERMOELASTIC COUPLING EFFECTS ON SUPERSONIC AIRCRAFT FLIGHT DYNAMICS

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ABSTRACT

Supersonic aircraft experience severe aerothermoelastic coupling phenomena that pose risks of structural failure and control system instability. In this study, a unified aerothermoelastic and flight dynamics formulation and analysis method for supersonic aircraft was developed.

A supersonic unsteady aerodynamic calculation method was established based on a three-dimensional discretization form of local piston theory (3D-LPT), which has both high efficiency and high accuracy while being suitable for complex 3D configuration analysis:

$$p = p_L + \rho_L a_L \boldsymbol{n}^{\mathrm{T}} \left(\frac{\partial \boldsymbol{w}}{\partial t} + V_{Lx} \frac{\partial \boldsymbol{w}}{\partial x} + V_{Ly} \frac{\partial \boldsymbol{w}}{\partial y} + V_{Lz} \frac{\partial \boldsymbol{w}}{\partial z} \right) \tag{1}$$

In the body axis system *Oxyz*, $\boldsymbol{n}$ is the outward normal unit vector of a wall surface element and $\boldsymbol{w}$ denotes the displacements of the element. V_{Lx} , V_{Ly} and V_{Lz} are the projection of surface local flow velocity V_L in the *x*, *y*, and *z* directions, respectively. ρ_L and a_L are the local density and sound speed, respectively. These local flow quantities are determined by CFD calculation.

Based on thermal mode theory, a one-way coupled thermo-structural dynamic model was developed. This study employs the CFD method to calculate the surface temperature distribution of the initial configuration, which is then applied as the boundary condition for structural heat conduction analysis to obtain the steady structural temperature field.

The linear stiffness matrix with thermal effects is defined as follows:

$$\boldsymbol{K}_{lT} = \int_V \boldsymbol{B}^T \boldsymbol{D}_T \boldsymbol{B} dV \tag{2}$$

where V represents the structural integration domain, $\boldsymbol{B}$ is the geometry matrix, and $\boldsymbol{D}_T$ denotes the temperature-dependent elastic matrix related to elastic modulus and Poisson's ratio.

The effect of thermal stress can be represented by an initial stress stiffness matrix:

$$\boldsymbol{K}_\sigma = \int \boldsymbol{G}^T \boldsymbol{s} \boldsymbol{G} dV \tag{3}$$

where $\boldsymbol{G}$ is the shape function matrix, and $\boldsymbol{s}$ is the thermal stress matrix.

The thermal stiffness matrix is the combination of the linear stiffness matrix with thermal effects and the thermal stress stiffness matrix:

$$\boldsymbol{K}_T = \boldsymbol{K}_{lT} + \boldsymbol{K}_\sigma \tag{4}$$

Structural thermal modes can be obtained from the structural mass matrix and the structural total thermal stiffness matrix. Structural dynamic analysis based on thermal modes effectively

captures the dynamic characteristics under a specific temperature field, allowing for aerothermoelastic modeling and analysis.

Based on the mean axis coordinate system, this study established a flight dynamics model for elastic aircraft that incorporates aerothermoelastic effects using the energy method, and further developed analytical frameworks for trim analysis and stability assessment of supersonic aircraft.

By introducing the rigid-body translational mode $\boldsymbol{\Phi}_t$ and rotational mode $\boldsymbol{\Phi}_r$, the flight dynamics equations for the elastic aircraft can be obtained by coupling the dynamic equations with the kinematic equations:

$$\begin{cases} \dot{\boldsymbol{V}} = -\tilde{\boldsymbol{\omega}}\boldsymbol{V} + \boldsymbol{\Phi}_{\mathrm{t}}^{\mathrm{T}}\left(\boldsymbol{f}_{\mathrm{G}} + \boldsymbol{f}_{\mathrm{A}} + \boldsymbol{f}_{\mathrm{T}}\right)/M \\ \dot{\boldsymbol{\omega}} = -\boldsymbol{J}^{-1}\tilde{\boldsymbol{\omega}}\boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{J}^{-1}\boldsymbol{\Phi}_{\mathrm{r}}^{\mathrm{T}}\left(\boldsymbol{f}_{\mathrm{G}} + \boldsymbol{f}_{\mathrm{A}} + \boldsymbol{f}_{\mathrm{T}}\right) \\ \boldsymbol{M}_e\ddot{\boldsymbol{q}}_e + \boldsymbol{B}_e\dot{\boldsymbol{q}}_e + \boldsymbol{K}_e\boldsymbol{q}_e = \boldsymbol{\Phi}_e^{\mathrm{T}}\left(\boldsymbol{f}_{\mathrm{G}} + \boldsymbol{f}_{\mathrm{A}} + \boldsymbol{f}_{\mathrm{T}}\right) - \boldsymbol{M}_{ec}\ddot{\boldsymbol{q}}_c \\ \dot{\boldsymbol{R}}_g = \boldsymbol{L}^{\mathrm{T}}\boldsymbol{V} \\ \dot{\boldsymbol{\Theta}} = \boldsymbol{D}^{-1}\boldsymbol{\omega} \end{cases} \tag{5}$$

where $\boldsymbol{\Phi}_e$ denotes the elastic modes. $\boldsymbol{f}_{\mathrm{G}}$, $\boldsymbol{f}_{\mathrm{A}}$ and $\boldsymbol{f}_{\mathrm{T}}$ is the load caused by gravity, aerodynamic force, and propulsion, respectively. $\boldsymbol{M}_e$, $\boldsymbol{B}_e$, $\boldsymbol{K}_e$ and $\boldsymbol{M}_{ec}$ is the generalized matrix of mass, damping, and stiffness, respectively, while $\boldsymbol{M}_{ec}$ is the mass coupling matrix between the elastic modes and the control surface deflection modes.

The static aeroelastic trim residual can be derived from the flight dynamics equations under the assumption that the time derivatives of elastic deformation terms can be neglected:

$$\begin{cases} \boldsymbol{s}_t = \boldsymbol{\Phi}_{\mathrm{t}}^{\mathrm{T}}\left(\boldsymbol{f}_{\mathrm{G}} + \boldsymbol{f}_{\mathrm{A}} + \boldsymbol{f}_{\mathrm{T}}\right) - M\dot{\boldsymbol{V}} - M\tilde{\boldsymbol{\omega}}\boldsymbol{V} \\ \boldsymbol{s}_r = \boldsymbol{\Phi}_{\mathrm{r}}^{\mathrm{T}}\left(\boldsymbol{f}_{\mathrm{G}} + \boldsymbol{f}_{\mathrm{A}} + \boldsymbol{f}_{\mathrm{T}}\right) - \boldsymbol{J}\dot{\boldsymbol{\omega}} - \tilde{\boldsymbol{\omega}}\boldsymbol{J}\boldsymbol{\omega} \\ \boldsymbol{s}_e = \boldsymbol{\Phi}_{\mathrm{e}}^{\mathrm{T}}\left(\boldsymbol{f}_{\mathrm{G}} + \boldsymbol{f}_{\mathrm{A}} + \boldsymbol{f}_{\mathrm{T}}\right) - \boldsymbol{K}_{\mathrm{e}}\boldsymbol{q}_{\mathrm{e}} \end{cases} \tag{6}$$

Due to the presence of nonlinear terms, the trim solution generally requires an iterative method to obtain.

Applying the small-perturbation assumption to Eq. (5) and eliminating the trim loads yields the linearized small perturbation equation:

$$\begin{cases} \Delta\dot{\boldsymbol{V}} + \tilde{\boldsymbol{\omega}}_0\Delta\boldsymbol{V} - \tilde{\boldsymbol{V}}_0\Delta\boldsymbol{\omega} = M^{-1}\boldsymbol{\Phi}_t^{\mathrm{T}}\left(\Delta\boldsymbol{f}_{\mathrm{G}} + \Delta\boldsymbol{f}_{\mathrm{A}} + \Delta\boldsymbol{f}_{\mathrm{T}}\right) \\ \boldsymbol{J}\Delta\dot{\boldsymbol{\omega}} + \left[\tilde{\boldsymbol{\omega}}_0\boldsymbol{J} - \widetilde{\left(\boldsymbol{J}\boldsymbol{\omega}_0\right)}\right]\Delta\boldsymbol{\omega} = \boldsymbol{\Phi}_r^{\mathrm{T}}\left(\Delta\boldsymbol{f}_{\mathrm{A}} + \Delta\boldsymbol{f}_{\mathrm{T}}\right) \\ \boldsymbol{M}_e\Delta\ddot{\boldsymbol{q}}_e + \boldsymbol{B}_e\Delta\dot{\boldsymbol{q}}_e + \boldsymbol{K}_e\Delta\boldsymbol{q}_e = \boldsymbol{\Phi}_e^{\mathrm{T}}\left(\Delta\boldsymbol{f}_{\mathrm{A}} + \Delta\boldsymbol{f}_{\mathrm{T}}\right) - \boldsymbol{M}_{ec}\Delta\ddot{\boldsymbol{q}}_c \\ \Delta\dot{\boldsymbol{R}} = \boldsymbol{L}_1\Delta\boldsymbol{V} + \boldsymbol{L}_2\Delta\boldsymbol{\Theta} \\ \Delta\dot{\boldsymbol{\Theta}} = \boldsymbol{D}_1\Delta\boldsymbol{\omega} + \boldsymbol{D}_2\Delta\boldsymbol{\Theta} \end{cases} \tag{7}$$

where $\boldsymbol{L}_1$, $\boldsymbol{L}_2$, $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$ are obtained by performing a first-order Taylor expansion of the original nonlinear kinematic equations about the trimmed state.

Eq. (7) can be transformed into state-space form:

$$\dot{\boldsymbol{x}}_{\mathrm{sdt}} = \boldsymbol{A}_{\mathrm{sdt}}\boldsymbol{x}_{\mathrm{sdt}} + \boldsymbol{B}_{\mathrm{sdt}}\boldsymbol{\eta}_{\mathrm{sdt}} \tag{8}$$

where $\boldsymbol{x}_{\mathrm{sdt}}$ is the state vector, $\boldsymbol{A}_{\mathrm{sdt}}$ is the state matrix, $\boldsymbol{\eta}_{\mathrm{sdt}}$ is the control input vector, and $\boldsymbol{B}_{\mathrm{sdt}}$ is the control input matrix.

The flight dynamic characteristics can be obtained through eigenvalue analysis of the system state matrix $\boldsymbol{A}_{\mathrm{sdt}}$, thereby providing the theoretical basis for the assessment of stability and flying qualities.

Component-level wind tunnel supersonic flutter testing was performed to validate the fluid-structure coupling component of the method's framework. With a relative error of the flutter speed being within 0.1%, the effectiveness of this method was preliminarily proved.

A full-process application of the framework proposed in this paper was conducted using a supersonic missile test model with a representative configuration.

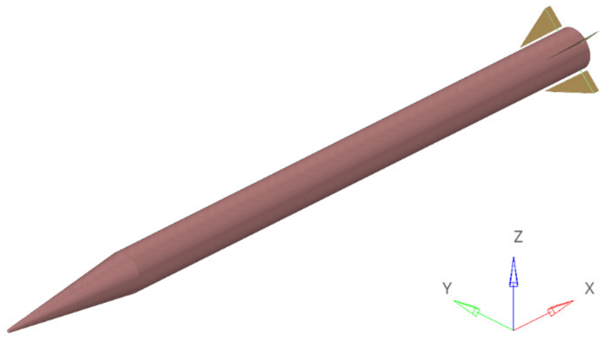


Figure 1: Test model configuration.

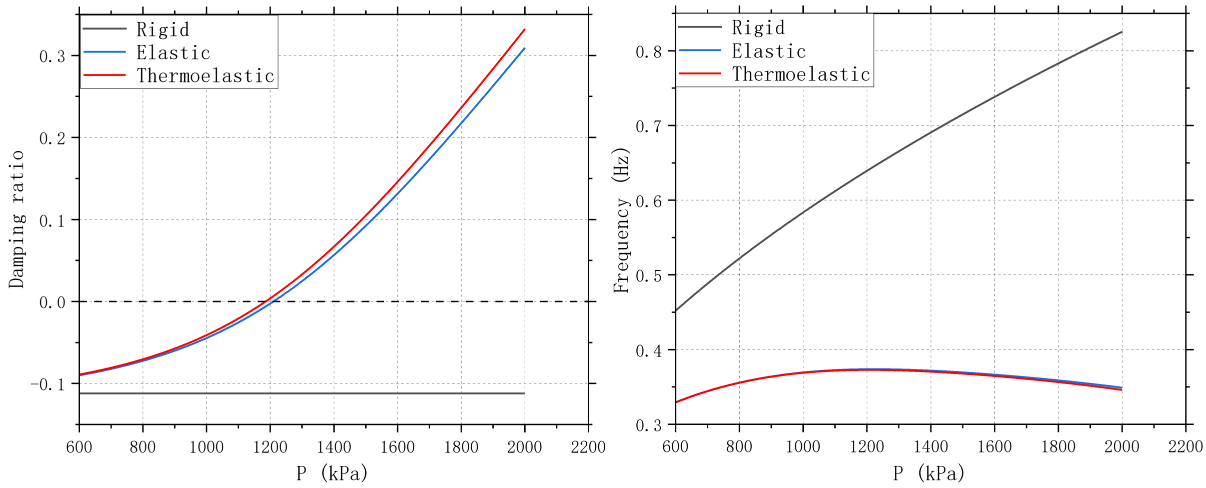
Thermo-structural dynamic analysis, static aerothermoelastic trim, flight dynamic characteristics, and stability analysis with thermal effects were performed.

Preliminary analysis results indicate that elastic effects have a significant influence on the flight dynamic modal characteristics and system stability, while thermal effects further exacerbate this influence.

Table 1: Comparisons of flight dynamic modal eigenvalues.

Coupling states	Short-period			Long-period		Phugoid	
	Eigenvalue	Damped frequency (Hz)	Damping ratio	Eigenvalue	Half-life (s)	Eigenvalue	Half-life (s)
Rigid	-0.3599±2.9612i	0.4713	0.1207	-5.8271×10^{-3}	118.9523	-0.2506	2.7660
Aeroelastic	-0.2356±2.1449i	0.3414	0.1092	-5.8340×10^{-3}	118.8117	-0.5053	1.3718
Aerothermoelastic	-0.2317±2.1349i	0.3398	0.1079	-5.8353×10^{-3}	118.7852	-0.5154	1.3449

Longitudinal bending and the control surface deflection modes reduce the stability of the flight dynamics short-period mode, while the phugoid mode exhibits significant changes in its dynamic characteristics and a notable reduction in its half-life.



(a) V-G diagram (b) V-f diagram

Figure 2: Flutter diagrams.

Elastic coupling induces flutter in the short-period mode, which was originally stable over a wide range of dynamic pressures, while thermal effects further reduce the flutter dynamic pressure.

Through the aforementioned analysis process, the modelling and analysis method proposed in this study has been preliminarily verified to be an efficient way to perform trim, flight dynamic characteristics, and stability analysis for a flexible supersonic aircraft under aerodynamic heating.