INITIALISATION STRATEGIES FOR AERODYNAMIC INSTABILITIES USING TIME SPECTRAL METHOD IN A NEW GENERATION FLOW SOLVER

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ABSTRACT

Limit-cycle oscillations (LCO), which arise from aerodynamic instabilities such as vortex shedding and shock buffet, and their aeroelastic effects are well studied in aerospace literature. For instance, the wake behind a circular cylinder becomes unstable at a Reynolds number of approximately 47 [Qu et al. [2013]]. This instability leads to the formation of the von Kárman vortex street, where alternating positive and negative vorticity values appear on both sides of the wake centreline. Shock buffet, on the other hand, is a transonic instability that triggers shock oscillations over an aerofoil. Two types of shock buffet exist: type I occurs over symmetric aerofoils at zero degrees angle of attack with the shocks oscillating on both the upper and lower surfaces with a 180° phase shift; type II occurs on supercritical aerofoils with the shock oscillating about a point typically on the suction side [Giannelis et al. [2017]].

An effective simulation method for an LCO is the time spectral method (TSM) [e.g. Mundis and Mavriplis[2017]], as it avoids computing initial flow transients. Owing to its spectral accuracy and coupling between all the time instances, fewer points per period are required. Consider the governing equation

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \mathbf{F}(\mathbf{u}, \nabla \mathbf{u}) + \mathbf{Q} = 0, \tag{1}$$

where $\partial \mathbf{u}/\partial t$ is the temporal derivative of the state variables $\mathbf{u}$, $\mathbf{F}(\mathbf{u}, \nabla \mathbf{u})$ represents the convective and viscous fluxes, and $\mathbf{Q}$ is any source term (e.g. resulting from turbulence modelling). In the TSM, assuming periodicity with period T, discrete Fourier and inverse Fourier transforms are used to discretise the governing equations in time. To prevent odd-even decoupling, an odd number of time instances, N_T, is employed [Gopinath and Jameson [2006]]. The number of time instances relates to the number of harmonics, N_H, through $N_T = 2N_H + 1$. Using the TSM, the discretised form of eqn. 1 is:

$$\sum_{j=0}^{N_T-1} d_{nj}\mathbf{u_j} + \mathbf{R}(\mathbf{u_n}) = 0, \qquad n = 0,1,2,\dots,N_T - 1 \tag{2}$$

where $\mathbf{R}(\mathbf{u_n})$ is the spatial residual in the n^{th} time instance, and

$$d_{nj} = \begin{cases} \frac{\pi}{\mathrm{T}}(\text{-}1)^{n-j}\csc\left(\frac{\pi(n-j)}{N_T}\right), n \neq j \\ 0, n = j \end{cases} \tag{3}$$

is the time derivative operator with odd numbers of time instances. This resembles the steady-state problem, routinely solved with industrial computational fluid dynamics capability, with the extra time derivative operator coupling all the time instances.

The time period of the flow is not known a-priori for an LCO. Instead, it can be obtained by using gradient descent to minimise the magnitude of the left-hand side of eqn. 2 [Gopinath and Jameson [2006]]. Let I_n^l be equal to the left-hand side of eqn. 2 at the iteration l. By computing $\partial\left(I_n^l\right)^2/\partial T$ and averaging it over all its components, an update of the time period is obtained:

$$T^{l+1} = T^l - \Delta T \frac{\partial\left(I_n^l\right)^2}{\partial T}, \tag{4}$$

where ΔT is a chosen constant. When the minimum is reached, the gradient is negligible and$T^{l+1} \approx T^l$.

The present work uses the new generation CFD software by ONERA, DLR and Airbus (CODA). CODA is the computational fluid dynamics (CFD) software being developed as part of a collaboration between the French Aerospace Lab ONERA, the German Aerospace Center (DLR), Airbus, and their European research partners. CODA is jointly owned by ONERA, DLR and Airbus. CODA employs discontinuous Galerkin and finite-volume methods for spatial discretisation on unstructured meshes, and it supports both implicit and explicit schemes for time integration. A GPU-compatible linear solver package, called Spliss [Krzikalla et al.[2021]], is used, which provides a range of linear solvers and preconditioners. The present work integrates the time period update algorithm (eqn. 4) into CODA's TSM framework [Haupt and Stück [2024]].

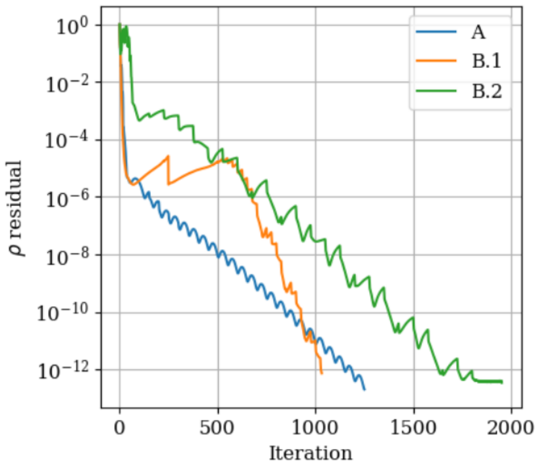
Unlike initialisation in initial value problems, where only the spatial field needs to be initialised at time $t = 0$ together with an appropriate choice of the time-step size, in TSM the time instances need to be equidistantly positioned and initialized in time appropriately for the problem at hand, which is often chosen to be reference freestream values. However, this can lead to problems when the system is prone to self-excited fluid instabilities. The initialisation methods for LCO simulations used in the present work are as follows:

- **Approach A (Nonlinear forced-motion):** This approach consists of two stages. First, the field is initialised to the freestream condition. The geometry is then harmonically excited (e.g. through a plunging or pitching motion) for N_p user-defined iterations at a specified amplitude and frequency, with these parameters typically selected based on physical insight and experience. This motion introduces a perturbation that prevents the subsequent stage from converging to the steady-state solution. For CODA, which has a strong implicit method implemented, the solution is able to attain the trivial steady-state solution by driving the derivative operator efficiently to zero. In the second stage, the structural forcing is stopped, and the time period is updated using eqn. 4 after every N_{upd} user-defined iterations.
- **Approach B (Linear response):** The response of the linearised governing equations to small amplitude harmonic forcing or disturbance can be used to introduce a perturbation in the initial field. If $\hat{\mathbf{u}}$ is the linear response and $\widehat{\mathbf{u}^*}$ is its complex conjugate, then the initial field is

$$U_n = U_{\mathrm{base}} + s\left\{\hat{\mathbf{u}}\ \exp\left(\frac{i2\pi n}{TN_T}\right) + \hat{\mathbf{u}}^*\ \exp\left(-\frac{i2\pi n}{TN_T}\right)\right\}, \qquad n = 0,1,\cdots,N_T - 1, \tag{5}$$

  where U_{base} is the base (or steady-state) solution, s is a scaling applied to the linear solution vector, and $i = \sqrt{-1}$. The two linearisation techniques used in this work are discussed below:

  1. **Unstable eigenmode to the linear stability analysis:** Linearising the governing equation about a base-flow solution and assuming exponential time evolution for the perturbation yields an eigenvalue problem. An LCO arises when a pair of complex conjugate eigenvalues crosses the imaginary axis (Hopf bifurcation). CODA solves the eigenvalue problem using the Krylov–Schur algorithm [Vevek et al. [2024]]. The complex eigenvector corresponding to the unstable eigenvalue, $\lambda = \omega_r + i\omega_i$, is $\hat{\mathbf{u}}$.
  2. **Linear frequency domain (LFD)**: The LFD solver in CODA [Vevek et al. [2022]] computes a linear aerodynamic response $\hat{\mathbf{u}}$ to a harmonic excitation from the linearised governing equations. This is a special case of approach A. It applies a small amplitude forcing of the form $\hat{\mathbf{x}}e^{i\omega t}$ with a user-defined amplitude and frequency. The resulting complex-valued aerodynamic response $\hat{\mathbf{u}}$ captures both the amplitude modulation and the phase difference relative to the harmonic forcing.



(a) (b)

Figure 1: Comparison of (a) convergence of all three approaches, and (b) lift coefficient of converged result.

For the sake of brevity, the current abstract details the comparison of the three approaches for vortex shedding behind a cylinder at Reynolds number 50 and Mach number 0.2, with 5 harmonics. Figure 1a shows the convergence history, based on the density residual, for all three approaches. It can be seen that approaches A and B.1 converge at approximately the same number of iterations, while approach B.2 requires a greater number of iterations. Figure 1b compares the lift coefficient for all three methods. The lift coefficients are in agreement with each other for all three approaches. The amplitude of the fluctuating lift coefficient (~ 0.04) and the Strouhal number ($\mathrm{St} = D/U_\infty T \approx 0.12$) is similar to that provided in the literature [Qu et al. [2013]], and found with the reference time-domain simulations herein. Apart from testing the efficiency of the method for vortex shedding at different Reynolds numbers, we also look at the effect of increasing the number of harmonics, which calls for strong implicit TSM solution strategies. The ability of these approaches to simulate shock buffet at different angles of attack and Mach numbers will also be tested in the final manuscript.

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