Limit Cycle Oscillation predictions using the Harmonic Balance Method and the nonlinear Source and Doublet Panel Method

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It is well known that nonlinear aeroelastic systems can undergo self-excited periodic responses, known as Limit Cycle Oscillations (LCO), which can be predicted numerically using time integration or faster and more efficient frequency domain techniques, such as the Harmonic Balance Method (HBM) (see for example Dimitriadis, 2008). The dynamics of any nonlinear autonomous system can be formulated in state space, and it is represented by the set of ordinary differential equations:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{\mu}) \tag{1}$$

where $\boldsymbol{x}$ are the system states, $\boldsymbol{\mu}$ the system parameters and $\boldsymbol{f}$ a nonlinear function. If the equations are real, we can assume a periodic solution for the state variable of the form:

$$\boldsymbol{x}(t) = \boldsymbol{x}_0 + \sum_{k=1}^{N} \boldsymbol{x}_{ks} \sin(k\omega_0 t) + \boldsymbol{x}_{kc} \cos(k\omega_0 t) \tag{2}$$

where ω_0 is the oscillation frequency, $\boldsymbol{x}_0$ is the steady response, $\boldsymbol{x}_{ks}$ are the coefficients of sinusoidal response and $\boldsymbol{x}_{kc}$ the coefficients of cosinusoidal response.

However, for aeroelastic systems the aerodynamic forces are conveniently expressed in the frequency domain, and the resulting equations of motion are therefore complex, so that equation (1) becomes

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{\mu}, \omega) \tag{3}$$

In this work, we demonstrate the intricacies of using the HBM and numerical continuation in nonlinear aeroelastic analysis of systems with frequency-domain aerodynamic loads. First, the methodology will be demonstrated on a typical aeroelastic section with nonlinear stiffness and Theodorsen aerodynamics (Theodorsen 1935) and then on a 3D wing with aerodynamic loads obtained from the nonlinear Source and Doublet Panel Method (Dimitriadis et al. 2025). In both cases, the aeroelastic equation of motion is written entirely in the frequency domain

$$i\omega \boldsymbol{x}(\omega) = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{\mu}, \omega) \tag{4}$$

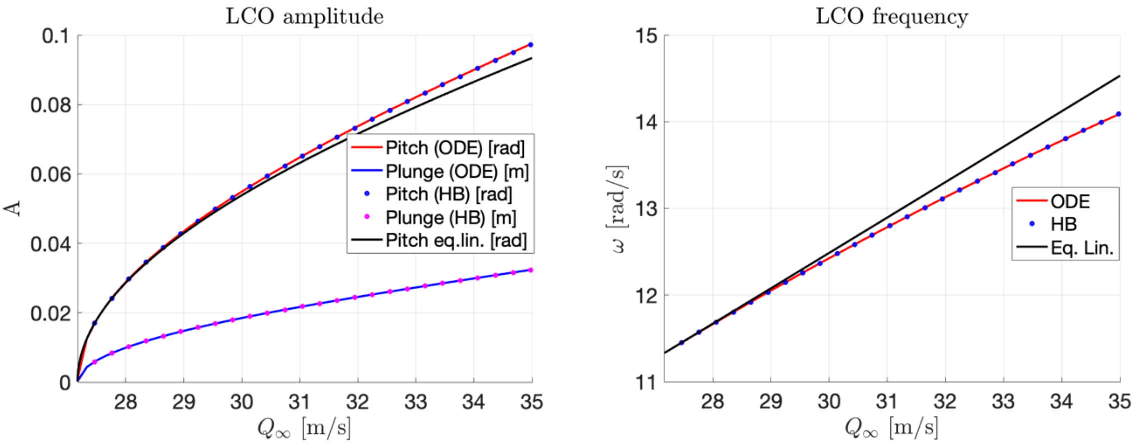
and the system's LCO response is selected as

$$\boldsymbol{x}(\omega) = \boldsymbol{x}_0 \delta(\omega) + \sum_{k=1}^{N} \boldsymbol{x}_k(k\omega_0)\, \delta(\omega - k\omega_0) + \sum_{k=1}^{N} \boldsymbol{x}_k^*(k\omega_0)\, \delta(\omega + k\omega_0)$$

where $\boldsymbol{x}(\omega)$, $\boldsymbol{x}_k(k\omega_0)$ and $\boldsymbol{x}_k^*(k\omega_0)$ are all complex and δ is the Kronecker Delta function, in vector notation, the *i*-th state $x_i(\omega)$ is written as

$$x_i(\omega) = \left(x_{k_i}^*(k\omega_0) \quad \cdots \quad x_{1_i}^*(\omega_0) \quad x_{0_i} \quad x_{1_i}(\omega_0) \quad \cdots \quad x_{k_i}(k\omega_0) \right)$$

Then, nonlinear terms, such as $x_i(t)^3$ can be evaluated in the frequency domain as the convolution $x_i(\omega) * x_i(\omega) * x_i(\omega)$, where the $*$ operator denotes convolution.



Comparison between time integration, Harmonic Balance and equivalent linearisation solution for a pitch-plunge wing with hardening cubic stiffness in pitch

Dimitriadis et al. (2025) show that the pressure coefficient on the surface can be calculated using second or higher order approximations for the unsteady Bernoulli equation. This calculation has already been implemented in the Source and Doublet Panel Method up to second order, but only the first order component is of interest in the prediction of flutter stability. For LCO prediction, the second and higher order terms are also of interest and therefore the SDPM becomes a nonlinear aerodynamic modelling method, even if the nonlinearity is weak and does not include contributions from viscous or transonic effects. It will be shown that the higher order pressure terms can be included in the aeroelastic equations, in the same way as nonlinear structural terms. Then, the resulting limit cycle predictions will include contributions from both the structure and the aerodynamics.

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