

Application of Udwadia-Kalaba method to flared folding wing tips: a low order explicit model for numerical continuation

James Ascham*, Mark Lowenberg†, Jonathan Cooper‡, Djamel Rezgui§

University of Bristol, Bristol, BS8 1TR, UK

To reduce the aviation industry’s environmental impact, civil aircraft are adopting longer, more flexible wings to improve induced drag, and hence reduce fuel burn. However, these extended wings can, consequentially, exceed airport infrastructure capabilities, such as gate or runway restrictions. Folding wing tips, which may be actuated inward during taxi, have been adopted by long-range wide-body aircraft. Beyond ground operations, deploying the wing tip during flight as a passive control surface has shown promise in both simulation and experimental testing. Such benefits include gust loads alleviation [2, 4], and improvements to roll characteristics [5]. However, the high flexibility of modern aircraft wings raise uncertainty about how the folding wing tip system’s nonlinear dynamics and stability will behave across large operating and parameter ranges, particularly when coupled with the full aircraft dynamics.

Recent work concerned with the dynamics of flared folding wing tips have either focused on a reduced coordinates approach to modelling the geometry of the folding wing tip, using simplifying linear assumptions [4] or have sought to capture the full geometric nonlinearity, consequently producing high-order, implicit or differential algebraic equations prohibitive to numerical continuation [3]. In comparison, this study employs the multibody dynamics method of [Udwadia and Kalaba](#) [7]; in this way, a minimal state set of ordinary differential equations, permissible to numerical continuation, is produced.

In this work, we aim to explore a low-order multibody modelling approach, to better predict and characterise nonlinear aeroelastic phenomena in FFWT systems. In Figure 2, we show validation of the proposed aeroelastic model in comparison to experimentally derived data, and a bifurcation diagram of fold angle against freestream velocity.

In the final paper, a detailed description of the full model, Figure 1, is to be provided. The inner wing is approximated via a Rayleigh-Ritz beam model, and the wing tip considered a rigid point mass, where the full geometric nonlinearity of both the fluid-structure interface, and the flared hinge line is retained. Validation of the rigid multibody dynamics is achieved by comparison to a flared double pendulum modelled using the Absolute Nodal Coordinate Formulation (ANCF), and corrected for numerical drift introduced by time integration [1]. A folding wing tip experimental model is used as the validation test case for the full aeroelastic system, where comparison can be made against both experimental and numerical data. Due to large reference rotations of the wing tip influencing drastically the spanwise lift distribution, the model is coupled to an unsteady lifting line method proposed by [Jayatilake et al.](#) [6], and modified to account for the change in line geometry. Bifurcation analysis is performed on the full aeroelastic model, showing good agreement with both numerical and experimental data.

*Research Associate, School of Civil, Aerospace and Design Engineering

†Professor of Flight Dynamics, School of Civil, Aerospace and Design Engineering.

‡Airbus RAEng Sir George White Professor of Aerospace Engineering, School of Civil, Aerospace and Design Engineering, FAIAA

§Associate Professor in Aerospace Engineering, School of Civil, Aerospace and Design Engineering.

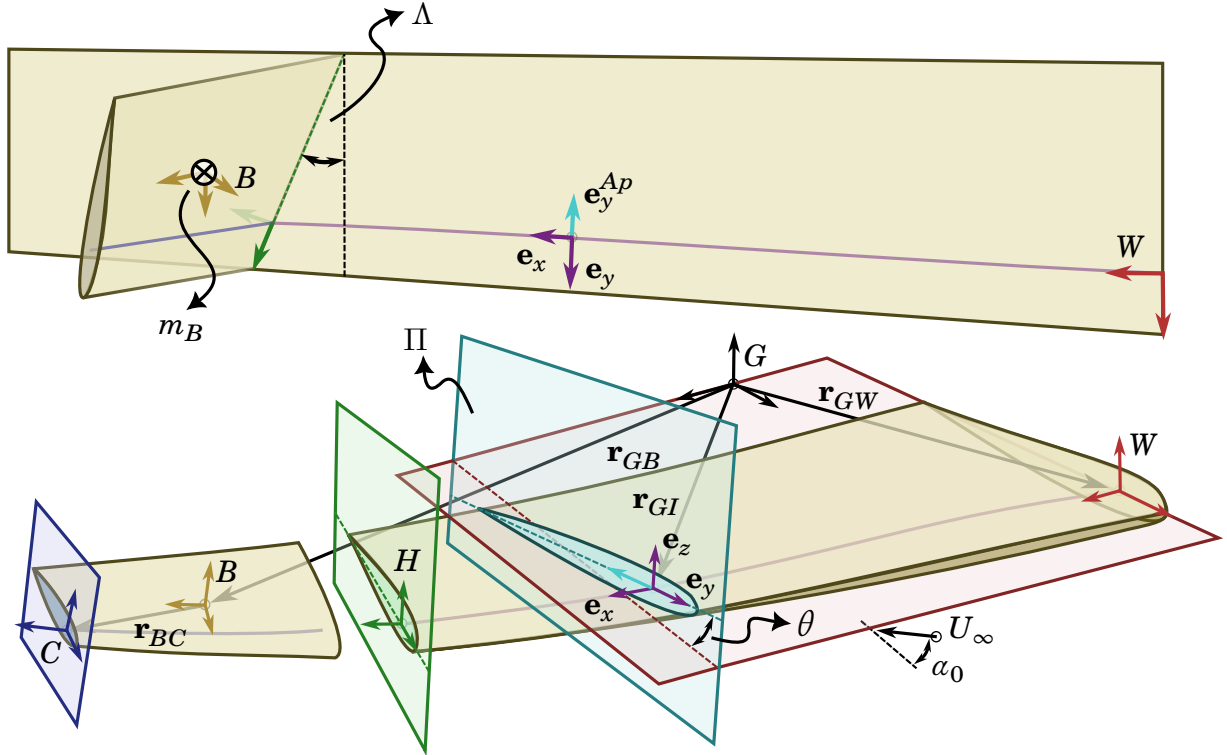
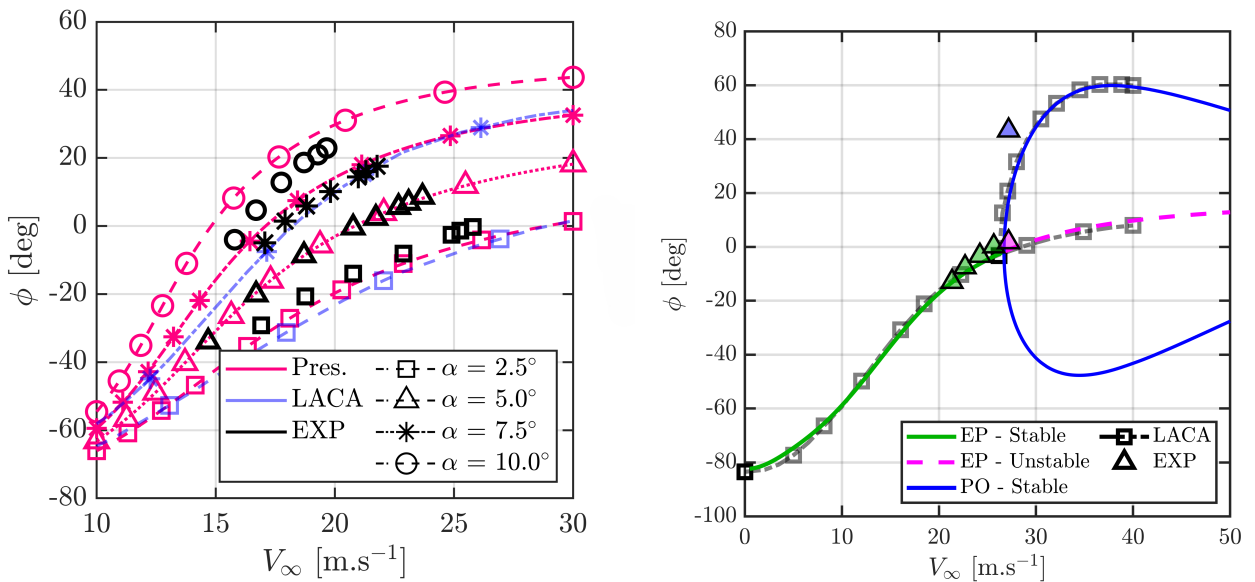


Figure 1: Geometric definitions of the FFWT, structural deformation, and aerodynamic coordinate systems.



(a) Static equilibrium for a range of angle of attack. (b) Bifurcation diagram. Angle of attack = 2.5 [deg].

Figure 2: Fold angle of the wing tip multibody model, in comparison to experimental data.

Bibliography

- [1] D. J. Braun and M. Goldfarb. Simulation of constrained mechanical systems — part i: An equation of motion. *Journal of Applied Mechanics*, 79(4):041017, 05 2012. ISSN 0021-8936. doi: 10.1115/1.4005572. URL <https://doi.org/10.1115/1.4005572>.
- [2] A. Castrichini, T. Wilson, F. Saltari, F. Mastroddi, N. Viceconti, and J. E. Cooper. Aeroelastics flight dynamics coupling effects of the semi-aeroelastic hinge device. *Journal of Aircraft*, 57(2):333–341, 2020. doi: 10.2514/1.C035602. URL <https://doi.org/10.2514/1.C035602>.
- [3] K. C.-W. Cheng, A. Cea, R. Palacios, A. Castrichini, and T. Wilson. Nonlinear multibody modeling of flexible aircraft with flared hinged wingtips. *Journal of Aircraft*, 62(4):929–948, 2025. doi: 10.2514/1.C038094. URL <https://doi.org/10.2514/1.C038094>.
- [4] R. C. M. Cheung, D. Rezgui, J. E. Cooper, and T. Wilson. Testing of folding wingtip for gust load alleviation of flexible high-aspect-ratio wing. *Journal of Aircraft*, 57(5):876–888, 2020. doi: 10.2514/1.C035732. URL <https://doi.org/10.2514/1.C035732>.
- [5] F. Healy, R. Cheung, T. Neofet, M. Lowenberg, D. Rezgui, J. Cooper, A. Castrichini, and T. Wilson. Folding wingtips for improved roll performance. *Journal of Aircraft*, 59(1):15–28, 2022. doi: 10.2514/1.C036372. URL <https://doi.org/10.2514/1.C036372>.
- [6] S. Jayatilake, M. Lowenberg, B. K. S. Woods, and B. Titurus. Nonlinear aeroelastic modelling and analysis of a geometrically nonlinear wing with combined unsteady sectional and lifting line aerodynamics. *Nonlinear Dynamics*, 113:14657–14693, 2025. doi: 10.1007/s11071-025-10936-4. URL <https://doi.org/10.1007/s11071-025-10936-4>.
- [7] F. E. Udwardia and R. E. Kalaba. On the foundations of analytical dynamics. *International Journal of Non-linear Mechanics*, 37(6):1079–1090, 2002.