DYNAMIC STABILITY OF SMART WING WITH MORPHING-INDUCED CYCLIC STIFFNESS VARIATION

Manoj Prabhakar*, Senthil Murugan

*Indian Institute of Technology Madras
Chennai 600036, India

ABSTRACT

Morphing or adaptive wings are recently being incorporated with smart structures capable of on-demand stiffness variation to reduce the energy consumption for morphing [1]. However, if the stiffness variation is performed periodically with time (cyclic), parametric instabilities can be induced in the wing structure - which is detrimental to the stability of morphing wing operation. Moreover, since the wing with aeroelastic bending-torsion coupling is a potentially self-excited system, the interaction between the flutter instability and parametric instability is important and well is not well understood.

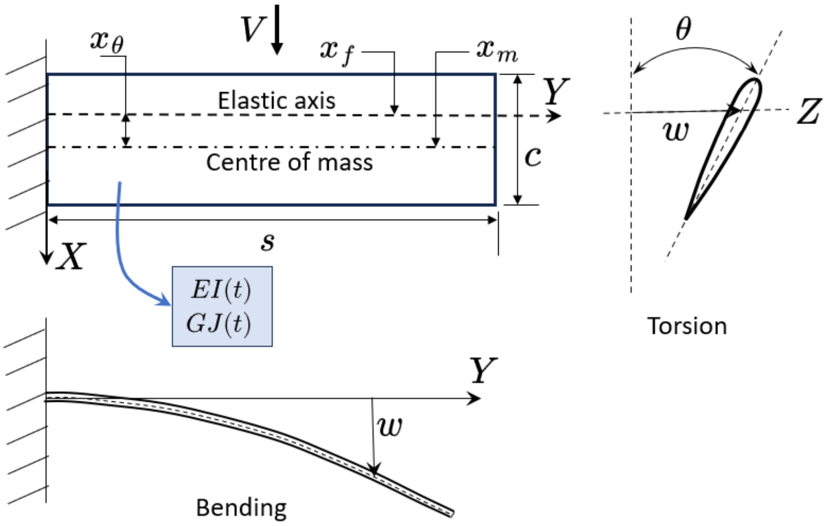

Figure 1: Flexible cantilever wing model with time-varying stiffness.

In this study, the parametric instability of a flexible cantilever wing with morphing-induced cyclic stiffness is investigated. The morphing-induced stiffness variation is modelled as time-periodic bending and torsional stiffness variation as

$$EI(t) = EI_0[1 + \varepsilon \cos(\Omega\, t)]$$

$$GJ(t) = GJ_0[1 + \varepsilon \cos(\Omega\, t)]$$

The stiffness are varied with equal magnitudes (ε) and equal excitation frequency (Ω) about their corresponding mean stiffness values.

The equation of motion for bending and torsion motion is given as

$$m\,\ddot{w} + mb(x_\theta - a)\ddot{\theta} + EI(t)\,w'''' = L_a$$
$$mb(x_\theta - a)\,\ddot{w} + I\ddot{\theta} - GJ(t)\,\theta'' = M_a$$

Where the aerodynamic loads are given in the right-hand side of the equation, which are functions of the airspeed V. The aerodynamic loads are obtained using the Theodorson's unsteady aerodynamics.

A **finite element model** is constructed by discretizing the wing structure into eight aeroelastic beam elements. Each beam element has eight degrees of freedom, including two aerodynamic lag states and six structural deformation states. The resulting equation of motion is in form of multi-dimensional Mathieu's equation with aeroelastic bending-torsion coupling, given as

$$\mathbf{M}_G\ddot{\mathbf{q}} + \mathbf{C}_G\dot{\mathbf{q}} + \mathbf{K}_G(t)\mathbf{q} = \mathbf{0}, \qquad (1)$$

Where **q** is the global nodal variable, and the subscript G indicates global matrices.

Stability analysis method:

The dynamic stability of equation (1) is investigated numerically using **Floquet theory** with the frozen time technique. Since the system has a large number of degrees of freedom (32 nodal variables), frozen time technique reduces the computation time significantly. Also, the main parameters of interest for the stability analysis are the magnitude of stiffness variation ε, excitation frequency Ω, and airspeed V. For every parameter set (ε, Ω, V), Floquet multipliers ρ_k are calculated. If any one of the $|\rho_k|$ values are > 1, the system is unstable as per Floquet theory.

Results:

The results from the stability analysis show that dynamic stability of the wing is significantly affected by morphing-induced cyclic stiffness variation. The stability and instability regimes are quantified using stability chart shown below

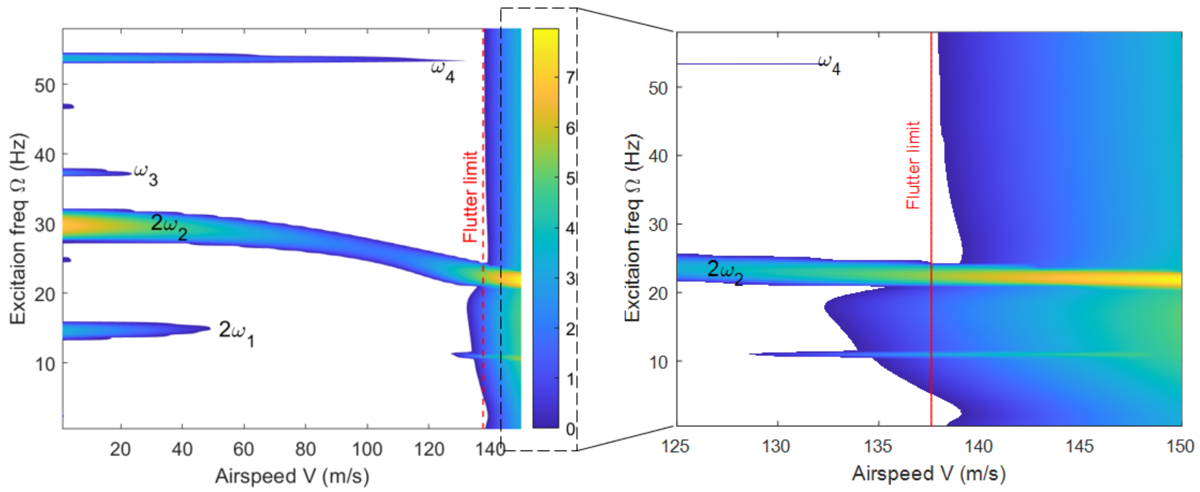

Figure 2: Instability chart in the (Ω, V) plane for $\varepsilon = 0.3$. The colored regions are unstable.

In the stability chart above, the wing flutter speed (flutter limit) at 137.6 m/s is shown as a red line. And the j^{th} natural frequency of the wing is denoted by ω_j, and they are strongly affected by airspeed. In the chart, for lower airspeeds, multiple parametric instabilities are observed for excitation frequencies around resonance frequencies: $2\omega_1, 2\omega_2, \omega_3, \omega_4$. Among them the instability corresponding to $2\omega_2$ is the widest and do not completely vanish with airspeed. Rather, it moves to lower Ω values until merging with a wide instability region existing beyond the flutter limit. Since ω_2 corresponds to a torsion-dominated mode, the stability of wing is strongly affected by torsion stiffness variation.

Effect of phase difference:

For applications where multiple variable stiffness smart structures are operated simultaneously in the wing, due to potential asynchronicity, phase difference between *EI(t)* and *GJ(t)* can occur.

$$EI(t) = EI_0[1 + \varepsilon \cos(\Omega\, t)]$$

$$GJ(t) = GJ_0[1 + \varepsilon \cos(\Omega\, t + \Phi)]$$

where, the phase difference of Φ is introduced between the variable stiffness.

The stability chart with $\Phi = \pi$ is shown below

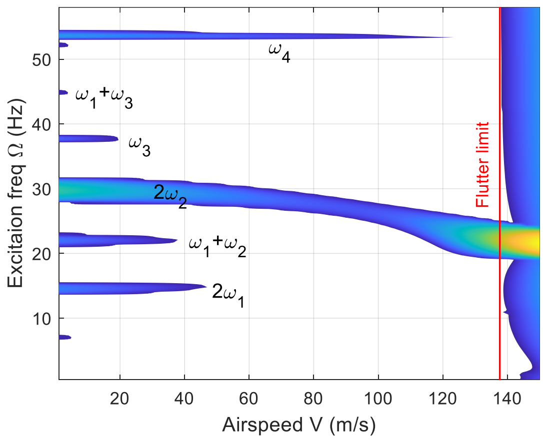

Figure 3: Instability chart in the (Ω, V) plane for $\varepsilon = 0.3$ and $\Phi = \pi$. The colored regions are unstable.

The chart shows emergence of new combination instability for resonant Ω around $\omega_1 + \omega_2$ and $\omega_1 + \omega_3$.

Conclusion:

The study shows that the dynamic stability of the wing is significantly affected by morphing-induced cyclic stiffness variation. The parametric instabilities are characterized by resonant frequencies that are functions of the modal frequencies of the wing. Further, due to interaction between self-excitation and cyclic stiffness, the wing exhibits complex stability behaviour for airspeeds close to flutter speed. Moreover, any phase difference between the bending and torsion stiffness variation cause emergence of new combination-type parametric instabilities.

The results are useful in the design and safe operation of variable stiffness mechanisms in morphing wing application.

In the full paper, extensive analysis of instability regions around the flutter limit will be presented. Moreover, the response behaviour in the identified stable regions will be investigated for morphing-induced stiffness variation effects.

Reference:

1. Kuder, I.K., Arrieta, A.F., Raither, W.E. and Ermanni, P., 2013. Variable stiffness material and structural concepts for morphing applications. *Progress in Aerospace Sciences*, *63*, pp.33-55.